

# Pre-Cal Test

November 19, 2011

- Which of the following four numbers is the largest?
  - $\log_2 3$
  - $\log_4 8$
  - $\log_3 2$
  - $\log_5 10$
- The point on the circle  $x^2 + y^2 + 6x + 8y = 75$  that is closest to the origin is at what distance from the origin?
- The complex number  $z$  satisfies  $z + |z| = 2 + 8i$ . What is  $|z|^2$ ?
- Given  $\log_9 X + \log_{12} Y = \log_{16}(X + Y)$ , evaluate  $\frac{Y}{X}$ .
- If  $2^{\sin^2 x} - 2^{\cos^2 x} = \cos(2x)$ , find the smallest positive value of  $x$ .
- How many real roots does  $\sin(x) = \log_{10} x$  have?
- Let  $x$  be a real number selected uniformly at random between 100 and 200, inclusive. If  $\lfloor \sqrt{x} \rfloor = 12$ , compute the probability that  $\lfloor \sqrt{100x} \rfloor = 120$ .
- Find the number of ordered pairs  $(x, y)$  that satisfy  $x^3 + 4x^2 + 4xy^2 - x = 2y$  if  $x$  and  $y$  are both integers between  $-100$  and  $100$  (inclusive).
- Alan has one “Judgment Dragon” card in his 4-card hand and 10 cards remaining in his deck. 1 of the 10 cards in his deck is “Judgment Dragon.” Each turn, he draws one card from his deck, and then discards one random card to Jonathan’s “Spirit Reaper.” After 3 of these draw-discard cycles, what is the probability that Alan holds at least one “Judgment Dragon” in his hand?
- Given that  $23!$  has  $m$  divisors and  $21!$  has  $n$  divisors, calculate  $\frac{m}{n}$ .