Geometry Test

November 19, 2011

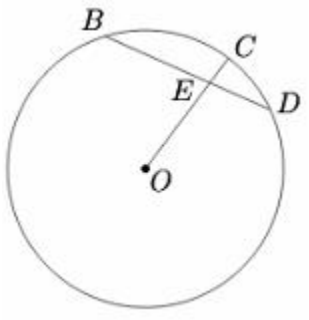
1. Two circles both of radii 8 have exactly one point in common. If X is one point on one circle, and Y is a point on the other circle, then what is the maximum possible length for line segment XY?

2. An equilateral triangle has the same area as a square. What is the ratio of a side of the square to the side of the triangle?

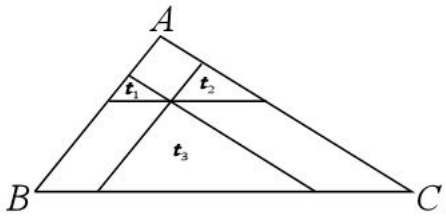
3. A and B are distinct points on a circle with center O. If $m\angle AOB = 72\,\text{deg}$, and length of arc $\overset{\frown}{AB}$ is 3π, what is the area of the sector AOB?

4. What is the area enclosed by the graph $|3x| + |4y| = 12$ in the xy-plane?

5. $\overline{DEB}$ is a chord of a circle such that $DE = 3$ and $EB = 5$. Let O be the center of the circle. Join OE and extend OE to cut the circle at C. Given that $EC = 1$, find the radius of the circle.



6. A point P is chosen in the interior of triangle ABC such that when lines are drawn through P parallel to the sides of ABC. The resulting smaller triangles t_1, t_2, and t_3 in the figure have areas 4, 9, and 49, respectively. Find the area of triangle ABC. (Figure is not to scale).



7. Triangle ABC has $AC = 15$ and $BC = 10$. Points D and E are located on AC and AB, respectively, so that $AD = CD$, and CE is the angle bisector of angle C. Let X be the point of intersection of BD and CE, and let Y be the point on line BD for which D is the midpoint of XY. If $AY = 6$, find EX.

8. Find the number of ordered pairs (x, y) that satisfy $x^3 + 4x^2 + 4xy^2 - x = 2y$ if x and y are both integers between -100 and 100 (inclusive).

9. Alan has one "Judgment Dragon" card in his 4-card hand and 10 cards remaining in his deck. 1 of the 10 cards in his deck is "Judgment Dragon." Each turn, he draws one card from his deck, and then discards one random card to Jonathan's "Spirit Reaper." After 3 of these draw-discard cycles, what is the probability that Alan holds at least one "Judgment Dragon" in his hand?

10. Given that 23! has m divisors and 21! has n divisors, calculate $\frac{m}{n}$.