

Calculus Test

November 19, 2011

1. Let $f(g(x)) = 12x + 9$. If, for some number a , $f'(a) = 4$, and $g(b) = a$, find $g'(b)$.
2. Let $r(\theta) = \ln[\sin(\theta)\cos(\theta)]$. Compute $r'(\frac{\pi}{6})$. Express the answer in simplest form such that there are no radicals in the denominator.
3. Let $f(x) = \frac{x^{2011}}{x+1}$. Find the number of trailing zeroes of the 2011th derivative of $f(-2)$. (*Trailing zeroes* are zeroes that go at the end of the number. For example, 140 has 1 trailing zero and 1,028,000,000 has 6 trailing zeroes.)
4. While studying for a calculus test, David accidentally spilled some ramen without realizing it. The soup creates a circular puddle on the floor spreading equally in all directions. The area is increasing at a constant rate of 4π cm² per second. In how many seconds after he spills the first drop will the soup touch a textbook 20 cm from the center of the puddle?
5. Let $h(x) = s = a\sqrt{(x^2 + 2xs + a^2(x^2 + 2xs + a^2(x^2 + 2xs + a^2(\dots))))}$, where a is a constant and $|a| < 1$. Calculate all possible values of $h'(x)$ in terms of a .
6. S is defined as the set of all z in the complex plane satisfying the inequalities

$$|z + 4 - 4i| \leq 4\sqrt{2} \text{ and}$$

$$|z - 4 - 4i| \geq 4\sqrt{2}.$$

Compute the area of S .

7. Patrick has a huge sheet of circular paper that he wants to use to make a snowflake. The paper has a radius of 10π , with center at point P . He folds the paper exactly in half 5 times so that the final, folded paper looks like a sector of the original circular sheet. He puts the paper down such that one of the straight edges is placed on the positive x -axis, with P at the origin. Then, on that edge, he cuts along the curve $y = |\sin^3 x \cos x|$ with x between 4π to 10π . Finally, he unfolds the paper. The area of the remaining paper has the form $A\pi^B - C$ where A , B , and C are integers. Evaluate $A + B + C$.

8. Find the number of ordered pairs (x, y) that satisfy $x^3 + 4x^2 + 4xy^2 - x = 2y$ if x and y are both integers between -100 and 100 (inclusive).
9. Alan has one “Judgment Dragon” card in his 4-card hand and 10 cards remaining in his deck. 1 of the 10 cards in his deck is “Judgment Dragon.” Each turn, he draws one card from his deck, and then discards one random card to Jonathan’s “Spirit Reaper.” After 3 of these draw-discard cycles, what is the probability that Alan holds at least one “Judgment Dragon” in his hand?
10. Given that $23!$ has m divisors and $21!$ has n divisors, calculate $\frac{m}{n}$.