Algebra II Test

November 19, 2011

1. Adam, Bob, and Charles are writing a math test together. If it takes Adam 60 hours to write it alone, 12 hours to write it with Bob, and 3 hours to write it with Bob and Charles, how many hours does it take Charles to write it alone?

2. Let p and q be prime numbers. If $p(q+1) = 330$, compute the sum of all possible q.

3. Let r, s, and t be the roots of $x^3 + 16x^2 + 27x - 252$. Find $r^2 + s^2 + t^2$.

4. Find the value of x for which there is a unique value of y if $y^2 + xy + 3y + x + 2 = 0$.

5. Let O be the center of a circle with radius r. $\overline{AC}$ and $\overline{BC}$ are tangents to the circle such that A, O, and B are collinear and B is on the circle. Let D be the point where $\overline{AC}$ and the circle intersect. If $BC = 12$ and the $AD = 8$, what is r?

6. How many ordered triples of positive integers (a, b, c) satisfy $\log_a b = c^{2011}$ and $a+b+c = 2011$?

7. If $i = \sqrt{-1}$, then $(1+i)^{200}$ can be expressed as $2^m + ni$, where m and n are real numbers. Find $m + n$.

8. Find the number of ordered pairs (x, y) that satisfy $x^3 + 4x^2 + 4xy^2 - x = 2y$ if x and y are both integers between -100 and 100 (inclusive).

9. Alan has one "Judgment Dragon" card in his 4-card hand and 10 cards remaining in his deck. 1 of the 10 cards in his deck is "Judgment Dragon." Each turn, he draws one card from his deck, and then discards one random card to Jonathan's "Spirit Reaper." After 3 of these draw-discard cycles, what is the probability that Alan holds at least one "Judgment Dragon" in his hand?

10. Given that 23! has m divisors and 21! has n divisors, calculate $\frac{m}{n}$.